Second-law bounds and a reproducible ledger for MHD energy recycling

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Abstract

We test whether reference-inlet exergy destruction can cap the additional credit in a hypersonic MHD energy ledger. A matched steady-flow exergy balance bounds improvement in exported work plus full outlet exergy by reference exergy destruction. It does not directly bound the earlier kinetic-equivalent credit: retained Joule energy and changes in residual thermal exergy also enter. At 220 K and 70% total-pressure recovery, destroyed exergy is 22.524 kJ/kg, whereas the corresponding ideal-nozzle kinetic loss is 23.712 kJ/kg. Published no-MHD recovery values at Mach 5, 8 and 10 give equivalent perfect-gas diagnostics of 9.89, 72.75 and 115.34 kJ/kg. Their comparison with a 263.2-kJ/kg ledger requirement is conditional on redefining all terms consistently as exergy. Equations, source arithmetic and independent checks are released. The reduced inlet remains unvalidated against a complete published inlet. This is a boundary and reproducibility note, with no claim of new propulsion physics.

1. Objective, ledger and notation

The question is whether an independently demonstrated inlet or cycle improvement can pay for energy extraction, imperfect electrical return and sustaining loads. Ionization affordability and MHD inlet control already appear in the literature [1-4]. A dimensionless map is not, by itself, a new physical result. Our objective is to expose which comparisons have a common thermodynamic boundary.

Per kg of processed inlet air, d denotes gross extracted stream energy, g its terminal-electrical fraction, e wall-plug ionization expenditure, a other electrical loads, s the fraction of remaining bus energy returned, h useful return conversion, and c a separately evaluated equivalent credit. Define r=g h s, x=e/d, alpha=a/d, chi=c/d and z=(e+a)/(g d). Then

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Delta = c-d+h s(g d-e-a) ; Delta/d = chi-1+r(1-z).
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Normalization requires d>0 and g>0; a self-powered bus needs 0<=z<=1 and compatible components satisfy 0<=r<=g<=1. The e/d form is Delta/d=chi-1+r-(r/g)(x+alpha), so an x-versus-r map still needs g and alpha. For r>0, x*=g[1+(chi-1)/r]-alpha. Equality is break-even; strict gain requires x<x*. Zero d or g requires the dimensional ledger; r=0 gives Delta/d=chi-1.

The historical illustration d=400 kJ/kg, g=h=0.60, s=1, a=12 kJ/kg and e=0 has r=0.36, z=0.05 and required chi=0.658, or c=263.2 kJ/kg. With c=0 its e*=-438.667 kJ/kg; with s=0.7 the required credit is 304.24 kJ/kg. These are assumed accounting cases, not measured engine limits. The same numerical parameters can illustrate an exergy ledger only after d, g, h and c have been defined consistently for that objective.

2. The second-law bound and why the credit needs care

Use environmental temperature T_e, pressure p_e and a common composition reference. T_e is not stagnation temperature T_t. For a steady adiabatic branch with common mass flow and inlet state, no added fuel or other external exergy, and negligible potential-energy difference, full flow exergy is psi=(h_static-h_e)-T_e(S-S_e)+u^2/2, where h_static denotes specific enthalpy, distinct from return efficiency h. With net electrical/shaft work W exported,

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W + psi_out = psi_in - T_e S_gen.
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Comparing modified m and reference b gives Delta(W+psi_out)=T_e(S_gen,b-S_gen,m)<=D_b, where D_b=T_e S_gen,b. If only selected useful products Y are counted, DeltaY=D_b-D_m-DeltaX_rejected at matched input exergy. A bound c_X<=D_b therefore applies to saved reference destruction alone, with other exergy transfers and rejected exergy matched. Fuel chemistry, changed combustion, heat exchange and nozzle/exhaust recovery must otherwise remain in the balance. An inlet loss budget cannot cap arbitrary whole-cycle improvements. The difference between ambient exergy destruction and a particular useful-energy loss has been discussed independently [5].

After ideal expansion to p_e, perfect-gas residual thermal exergy is X_th=cp[(T_x-T_e)-T_e ln(T_x/T_e)], and psi_out=E+X_th with E=u_x^2/2. The Faraday model extracts W=K d electrically and retains (1-K)d as Joule deposition. Its definition c_equiv=d+E_m-E_b gives

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c_equiv = (1-K)d + T_e(S_gen,b-S_gen,m) - (X_th,m-X_th,b).

c_equiv <= (1-K)d + D_b + X_th,b.
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This bound requires the same adiabatic, matched-pressure, perfect-gas comparison and nonnegative entropy generation and residual thermal exergy. The (1-K)d term is retained energy, not a new energy source. With free electrical overhead, Delta_proxy=T_e(S_gen,b-S_gen,m)-DeltaX_th-(1-h s)K d. Charging e+a adds -h s(e+a) only in the external-load approximation; plasma heating and chemistry remain unsolved. Counting whole-branch entropy reduction as c and subtracting generator losses again would double-count them.

For a passive no-work adiabatic perfect-gas reference, T_t is constant and Pi=p_t,out/p_t,in gives S_gen=-R ln(Pi). If T_in=T_e and expansion returns to inlet static pressure,

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D_b = T_e R ln(1/Pi) ; L_KE = cp T_e[Pi^(-(gamma-1)/gamma)-1].
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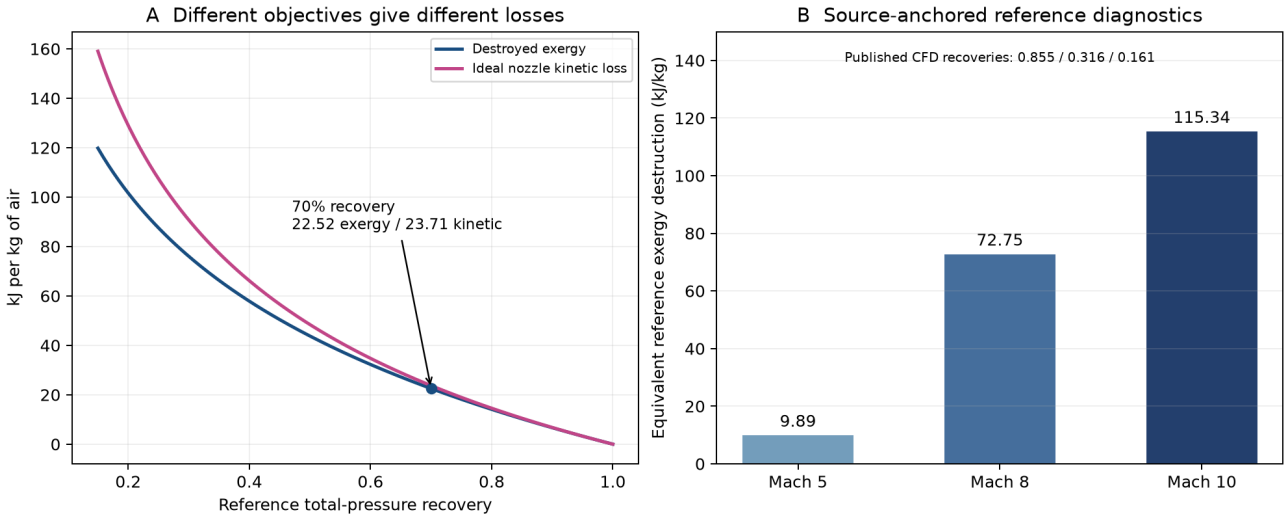
L_KE=D_b+X_th,b, strictly larger for Pi<1. At Pi=0.70, T_e=220 K, R=287.05 J/(kg K), gamma=1.4: S_gen=102.38354 J/(kg K), D_b=22.52438 kJ/kg, L_KE=23.71208 kJ/kg and X_th,b=1.18770 kJ/kg. Thus D_b is not a universal kinetic-loss cap. A Mach 5-to-2 deceleration adds no independent entropy term when T_t and Pi are specified; static pressure and temperature must change consistently.

3. Mach 5, 8 and 10: source-anchored diagnostics

Shneider et al. [4], Table 2, give no-MHD recoveries for a particular Mach-5-design four-ramp inlet. These computed values use stream-thrust-averaged throat states. We apply a 220-K dead state and a uniform/equivalent perfect-gas diagnostic. Taking a logarithm of an averaged recovery does not reconstruct a mass-flux entropy integral. At fixed Pi the formula has no independent Mach factor: Mach enters through recovery and gas state.

Flight Mach	Published Pi	D_X at 220 K	Conditional beta*
5	0.855	9.89 kJ/kg	26.61
8	0.316	72.75 kJ/kg	3.62
10	0.161	115.34 kJ/kg	2.28

Here beta* is required credit / D_b for a hypothetical consistently exergy-defined ledger with requirement 263.2 kJ/kg. Values above one exclude paying that requirement solely by avoiding the declared reference destruction. They do not reject the kinetic ledger or a reacting engine. Using source static temperatures 216.65, 222.70 and 225.63 K as the respective environments instead gives 9.74, 73.64 and 118.29 kJ/kg.



Perfect-gas screening at 220 K dead-state temperature. Panel A: matched ambient-pressure ideal expansion.
Panel B: Shneider et al. (2004), Table 2, equivalent averaged states. Neither panel bounds the existing kinetic credit c.

Figure 1. Original calculations. Panel A separates exergy destruction from ideal-nozzle kinetic loss at matched ambient pressure. Panel B shows geometry-specific equivalent diagnostics. Neither directly caps c_equiv. Source values and averaging conventions are included in the supplement.

At 220 K, a compatible inlet-only exergy requirement of 263.2 kJ/kg needs Pi<=exp[-263200/(220 R)] even if all reference destruction could be avoided. This is a conditional accounting screen, not an MHD operating threshold. A single normal shock gives much larger reference losses than the four-ramp inlet. High-Mach predictions also require variable properties and chemistry: perfect-gas T_t at Mach 8 and 10 would be 3036 and 4620 K. Active branches changing T_t need cp ln(T_t,out/T_t,in)-R ln(Pi), plus any boundary entropy-transfer terms.

4. Published-case comparison, screening labels and tests

Our scalar constant-area Faraday model followed by one normal shock cannot reproduce [4]'s coupled two-dimensional inlet, capture and plasma response. At Mach 5 its bare normal-shock recovery is 0.06172, versus the paper's complete-inlet 0.855; these are different boundary problems. The model is explicitly NOT VALIDATED against Shneider 2004. Importing the recovery or fitting sigma would not establish predictive validation.

A source arithmetic check is possible: the paper's Mach-10, K=0.5 energy partition closes to 0.0284% using rounded entries. This checks the reported Faraday split, not a flow prediction. HVEPS Test 3 [6] similarly gives 57.82^2*1.95=6.519147 kW at the external load. Ames velocity ratios [7] and pressure-based cycle efficiencies [8] use different observables; the accompanying efficiency note preserves that distinction.

The local model defines c_equiv(M_local,sigma,B;T,p,L,K,p_back). It sees sigma and B only as sigma B^2, enforces g=K and stops before its sonic singularity. It omits the conductivity/ionization-cost closure, Hall/sheath effects, real-gas chemistry, combustion, bleed and installed thrust. Its 27-case sweep tests the stated equations only; one interrupted case has no final credit.

We introduce Gamma_X=D_b/d and beta_save,crit=[1-r(1-z)]/Gamma_X as newly defined conditional exergy screening quantities. The latter is the fraction of baseline destruction that must be saved, not a Hall parameter or plasma stability threshold. It is undefined for D_b=0. At break-even beta<=1 is necessary under the compatible exergy-only assumptions, but not sufficient for a device. These labels replace ambiguous Gamma and beta_crit. B_controllable is reserved for a future newly defined field-screening domain; no numerical value or physical controllability bound follows from this ledger, and none is supplied.

Verification includes exact-rational accounting, scaling and bus-domain checks; passive-reference limits; kinetic/exergy counterexamples; and an independent reconstruction of full entropy and residual thermal-exergy balances for 15 reduced-flow states. Source arithmetic and invalid-input checks are separate. Scripts, figures, CSVs and the source-access matrix are supplied. No experimental confidence interval is assigned to assumptions. A different numeral base changes representation, not physical information.

The result is a conditional second-law statement and a reproducible demonstration that applying it to the previous c without translating the energy boundary is invalid. Improved combustion or nozzle performance requires a matched whole-cycle budget retaining chemical, thermal and rejected exergy. Further predictive validation requires common geometry, spatial plasma/flow inputs, matched outputs and an error metric against a published or measured case.

5. Prior art, research scope and release status

Prior studies already analyze MHD energy bypass, sustaining-ionization expenditure and inlet control [1-4]. This note claims neither their discovery nor a new propulsion mechanism. It offers an auditable distinction between an exergy-product bound and a kinetic-equivalent credit, with reproducible calculations; novelty in thermodynamics is not asserted.

Four journal texts remain needed for equation-level comparison: Riggins 2004 [9], Macheret et al. 2002 [10], Meyer et al. 2005 [11] and Lieu 2026 [12]. Missing full text is not evidence of absent overlap. The source matrix distinguishes author text inspected, metadata and unresolved retrievals. Equation-level comparison must be completed before submission readiness is claimed. Correspondence does not replace scientific validation.

DMGB, Independent researcher, is the author identity. No arXiv submission or endorsement is implied. Final author review, license and category/account eligibility remain necessary. Other chapter development is frozen until a preprint is actually released; a website working draft does not satisfy that condition. Earlier detailed methods remain in the supplement, with these v3 corrections taking precedence.

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